

# Modelling of damping characteristics in silicon-gold bilayer cantilevers

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This paper presents a simple method to extract the damping ratio of bilayer cantilevers using the static deflection profile of the released cantilevers. The extracted damping ratio is used to build a second order dynamical model of the cantilever. The model is validated by comparing the measured step response with the response predicted by the second order dynamical model.

## 1. Introduction

Cantilevers are the building blocks of various sensor and actuator systems [1]. They are used both in static and dynamic modes. In the dynamic mode, the cantilever is often vibrated at or near resonance and the change in resonance frequency in response to the physical quantity being measured is used for sensing. The sensitivity in measurement, i.e., the change in frequency per unit change in the physical quantity is critically dependent on the damping in the system. Damping represents energy dissipation in the system and is modelled by the damping ratio ( $\zeta$ ) in the second order dynamical model [2].

$$m\ddot{x} + d\dot{x} + kx \quad (1a)$$

$$\ddot{x} + 2\zeta\omega_o\dot{x} + \omega_o^2x \quad (1b)$$

In this work, the deflection profiles of silicon-gold bilayer cantilevers fabricated on silicon-on-insulator (SOI) substrates has been used to extract the damping ratio. Using the damping ratio and the resonance frequency, a dynamical model of the cantilever is built. The model is validated by comparing the measured and predicted step response of the system.

## 2. Samples

Figure.1 shows a schematic of silicon-gold bilayer cantilever considered in this work. The SOI wafer has a device layer thickness of 260nm and oxide layer thickness of 2 $\mu$ m. Three samples having gold layer thicknesses of 10nm, 20nm and 40nm have been thermally deposited on the device layer to form bilayer cantilevers. A deposition rate of 0.2nm/s was used for 10nm and 40nm gold layers. 20nm gold cantilevers were formed with a deposition rate of 0.5nm/s. Deflection profiles of four different cantilevers, two with 10nm gold layer thickness and one with 20nm and another one with 40nm gold layer thickness were used for the experiments. The cantilevers had a length (L) of 100 $\mu$ m and a width (W) of 20 $\mu$ m.

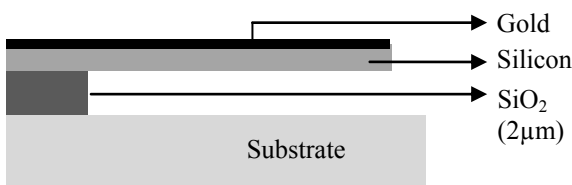


Fig:1: Schematic of the bilayer cantilevers

## 3. Extraction of damping ratio

The Si-Au cantilevers bend upwards due to the tensile stress present in the thermally deposited gold layer; the amount of deflection is related to the thickness of gold and deposition rate among other things. We chose cantilevers having four different static deflection profiles. The profiles were measured with an optical profilometer. The measured profiles,  $h(x)$ , are subsequently fitted by the following polynomial,

$$h(x) = ax^2 + bx + c, \quad (2)$$

where  $x$  is the distance along the length of the cantilever. The coefficients  $a$ ,  $b$  and  $c$  are estimated using the “polyfit” function in Matlab. The process of extracting and modelling is outlined in the following.

If the gap between the cantilever and the substrate is uniform throughout the length of the cantilever, the damping coefficient is given by [3],

$$d = \frac{\mu W^3 L}{g^3}, \quad (3)$$

where,  $g$  is the gap between the substrate and the cantilever. However, due to the residual stress gradients, the cantilevers deflect upwards making the air-gap non-uniform along the length of the cantilever. In this case, the air-gap can be modelled as a series of varying gaps as shown in figure 2. The damping coefficient for each of these units can be obtained from the equation 3 and the cumulative damping coefficient can be obtained by summing the damping ratio of all the individual elements, i.e.,

$$d = \sum_{n=1}^n \frac{\mu W^3 L_n}{g_n^3} \quad (4)$$

In the limit of infinite division units, the summation can be converted into an integral to obtain,

$$d = \frac{\mu W^3}{1} \int_0^L \frac{dL}{f(h)} \quad (5)$$

The polynomial in eqn.2 is used in  $f(h)$  to obtain smoothly varying gap

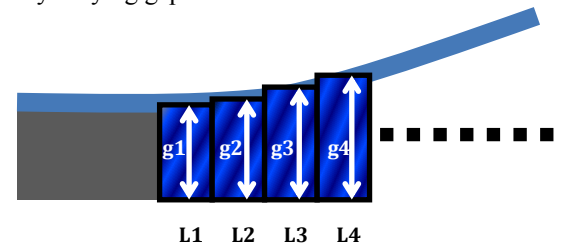


Fig:2: Non-uniform airgap

Figure.3 shows the measured (solid lines) deflection profiles of the fabricated cantilevers and the profiles

obtained using the “polyfit” function (dotted lines). Table 1 lists the all the coefficients of the polynomial and the corresponding damping ratio.

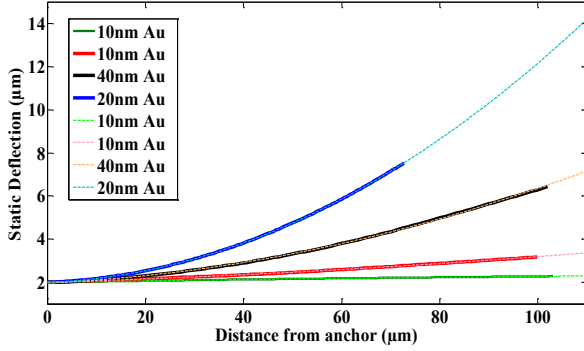


Fig.3: Measured (solid lines) and fitted (dotted lines) profiles

Table 1: Polynomial coefficients

Beam	Polynomial			Estimated $\zeta$
	a	b	c	
10Au (beam1)	51.315	0.007197	$2 \times 10^{-6}$	1.81
10Au (beam2)	-8.6146	0.0036032	$2 \times 10^{-6}$	6.54
20 Au	885.19	0.0079301	$2 \times 10^{-6}$	0.4470
40Au	372	0.01179	$2 \times 10^{-6}$	0.4846

#### 4. Model verification through step response

The deposition of gold on silicon alters the damping as well as the resonance frequency of the cantilever. Both parameters are important in determining the dynamic response. We estimated the resonance frequency of our bilayer cantilevers using the expression [4],

$$\left(\frac{f_{bi}}{f_s}\right)^2 = \left(\frac{(E_r t_r^3 + 1)(E_r t_r + 1) + 3E_r t_r (t_r + 1)^2}{(\rho_r t_r + 1)(E_r t_r + 1)}\right), \quad (6)$$

where,  $f_{bi}$  is the resonance frequency of the bilayer cantilever,  $f_s$  is the resonance frequency of the monolayer cantilever, and  $E_r$ ,  $t_r$  and  $\rho_r$  are the ratios of Young's moduli, thicknesses and densities respectively of the gold layers to silicon layer.

Based on the damping ratios and the resonance frequencies, we developed the second order dynamical model for the four cantilevers and simulated their step response using Simulink (Matlab). The simulated response was compared with the measured step response.

Step response of the cantilevers was measured by optically exciting the cantilevers with a 405nm laser source. Gold strongly absorbs this wavelength resulting in photothermal excitation of the cantilever [5]. The dynamics of the cantilevers was measured with a laser

vibrometer setup. Figure 4 shows the fabricated bilayer cantilever with actuation and sensing laser spots.



Fig.4: Actuation and sensing laser spots

Figure.5 shows the measured (solid lines) and simulated (dotted lines) step responses of the considered cantilevers, where we can observe general agreement between the measurement and simulation data.

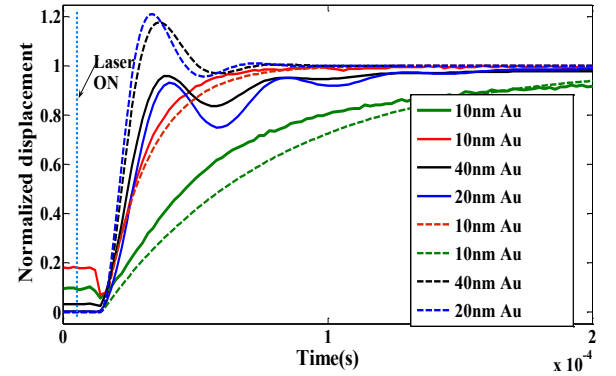


Fig.5: Measured (solid lines) and simulated (dotted lines) step response

#### 5. Conclusions

A simple method to extract the damping coefficient of bilayer cantilevers was presented. The method is general and can be applied to any cantilever system. Based on the extracted damping coefficient and estimated resonance frequency a second order dynamical model was constructed and validated.

#### References

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